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A COORDINATION GAME MODEL OF CHARITABLE GIVING AND SEED MONEY EFFECT

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Abstract: We point out that if potential donors for a charity project possess the warm-glow properties in their preferences, their behavior can be represented by a coordination game. Accordingly, we construct a simultaneous incomplete information game model of charitable giving on the basis of a simple global coordination game. We demonstrate that by considering the threshold shift effect of seed money exclusively, the proportion of donors and the total amount of donation strictly and continuously increase according to the amount of seed money. This result is quite compatible with the field experimental study of List and Lucking-Reiley [List, J. A., Lucking-Reiley, D., 2002. The Effects of Seed Money and Refunds on Charitable Giving: Experimental Evidence from a University Capital Campaign. Journal of Political Economy 110 (1), 215-233].

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1 Introduction

1.1 Charitable Giving and Coordination Game

Why should economists study charity? We present two answers to this question. One is the considerable impact of charitable behavior on our economy. Giving USA foundation (2006) estimates that in the United States, the total charitable donation for 2005 was \$260.28 billion, and \$199.07 billion (76.5% of the total) was contritubions from individuals. The size of these donations can constitute a topic of research. The National Center for Charitable Statistics (2006) reports that the total reported charitable deductions came to \$181.01 billion in 2005. It suggests that charitable giving also has an impact on the society via the tax deduction. The second answer is that charity is one of areas for which economists still do not possess a suitable theory. Andreoni (2006b) states "philanthropy is one of the greatest puzzles for economics." Mainstream economics has assumed that people are extremely selfish and are only interested in their self-beneficial consumptions. Economists have constructed models on the basis of this assumption. However, the assumption of selfishness is incompatible with charitable behavior. Even today, the field of philanthropy remains a puzzle and requires focused scholarly attention.

In recent years, it has been revealed that when considering charitable behavior, especially in large economies, the warm-glow property of preference is important. Preference with warm-glow property is typically represented by a utility function that possesses three dimensional domains: the private consumption, the total amount of donation from all contributors to a charity, and her own contribution to a charity; whereas classical preference based on public goods provision model possesses only two dimensional domains, the private consumption, and the total amount of donation from all contributors to a charity. Warm-glow is the property that increases a donor's utility as a result of her own contribution itself.[1] This property has been developed theoretically by Andreoni (1988, 1989, 1990), and supported empirically by studies such as Palfrey and Prisbrey (1996, 1997) and Ribar and Wilhelm (2002).[2] In particular, Andreoni (1988) and Ribar and Wilhelm (2002) indicate that in a large economy, the warm-glow property is crucial with regard to enacting adequate proportions of charitable contributions.

The simple simultaneous game of a capital campaign of charity as follows confirms their finding. Capital campaign constitutes a form of fund drives for charity that incurs a large fixed cost to ensure project success.[3] Campaigns to collect funds to built a hospital is a case in point. If the fixed cost for project success is huge and there are a large number of poten-

[1] Warm-glow is also called impure-altruism or joy-of-giving.

[2] Andreoni (2006b) contains a summary and discussions on the warm-glow property.

[3] We owe this definition of capital campaign to Andreoni (1998).

tial contributors, each contribution from each donor has almost no effect on the project success. Assume a potential donor in a large economy has two choices *Donate* and *Not Donate*, and possesses a simple classical preference that depends on her private consumption and whether the project is success or failure. The latter conditions is equivalent to whether the total amount of donation exceeds the fixed cost or not. Assume that even in the project failure, the donation is not refunded. Also assume that if a sufficiently large number of potential donors choose *Donate*, the project succeeds. Then her payoff is represented by

	Success	Failure
Donate	Happy	Unhappy
Not Donate	Very Happy	Neutral

Since a potential donor cares only for her private consumption and the project success and has no effect for the project success, she is better off choosing *Not Donate* if the project succeeds. Of course, *Not Donate* also yields greater happiness in the project failure. From this table, *Not Donate* is the strictly dominant strategy and each potential donor chooses this action. It is a prisoner's dilemma game and represents the classical free riding problem.

On the other hand, if people possess warm-glow property in their preferences, what is the payoff table in the same capital campaign game of a large economy? It is natural to consider that the payoff table is given by

	Success	Failure
Donate	Very Happy	Unhappy
Not Donate	Happy	Neutral

Contribution renders a donor with warm-glow property happier than not contributing if the project is success.[4] Then it has two pure strategy Nash equilibria. Either everybody chooses *Donate*, or everybody chooses *Not Donate*. This is a typical coordination game.

Coordination game is one of the most classical problems in game theory, and has significantly improved in recent years due to Carlsson and van Damme (1993). They consider a simultaneous incomplete information coordination game where each agent has little knowledge about the type distribution, and show the existence of a unique equilibrium with a little assumptions on the strategies of agents. Even in the standard Bayesian coordination game where type distribution is common knowledge, a unique equilibrium is difficult to obtain. Consequently, their study has drawn attentions. Their game model known as the *global game* becomes popular

[4]Ribar and Wilhelm (2002) find that in a large economy, the warm-glow property has much stronger effect than the altruistic property that is concerned with only the project success or failure in this example. Following their finding, the payoff to choose *Not Donate* at the project success is even represented by *Neutral*.

among game theorists and is extended in different aspects. Owing to Morris and Shin (1998, 2003), it is extended to encompass a coordination problem in a large economy.[5]

If there is a sufficiently large number of potential donors with the warm-glow property of preference, donation behavior for a charity project becomes a coordination game. In a situation of simultaneous donations, incompleteness of information on the valuation of each potential donor for the charity project is also appropriate. These are the reasons why we consider that global game is suitable to model charitable behavior. We construct a simultaneous incomplete information game model of charitable giving based on a simple global coordination game, and characterize a unique equilibrium of the game. By means of the comparative static analysis, we show that the model is compatible with the empirical studies of charitable behavior, especially the field experimental study of List and Lucking-Reiley (2002). We discuss this in the next subsection.

1.2 Seed Money

Seed money is a preliminary fund for charity that is publicly announced at the same time the project itself is announced. It is sometimes provided by leaders who are directly solicited to donate before the public announcement. It is well known among charity fund-raisers that if seed money is granted, the donation from general contributors increases even before Andreoni (1998) introduces this problem to public economists.

The field experimental study of List and Lucking-Reiley (2002) supports such knowledge of charity fund-raisers.[6] They divide 3,000 potential donors into six groups. To three groups with 500 potential donors each, they solicit the donation of a laboratory for environmental studies to purchase a computer with the fixed cost of \$3,000 with different seed money.[7] To the first group, they announce that seed contribution is \$300, to the second group, \$1000, and to the third group, \$2,000. The result is provided in Table 1. They clearly demonstrate that the total amount of contributions, the proportion of donors who actually contribute, and the per-capita amount of contribution strictly and continuously increase according to the amount of seed money.

[Table 1 enters around here.]

[5] Famous applications of the global game include the pricing debt problem (Morris and Shin, 2004) and bank runs (Goldstein and Pauzner, 2005). Both are typical coordination problems.

[6] Other empirical studies on seed money include List and Rondeau (2003) and Potters *et al.* (2005).

[7] The other three groups of potential donors are allocated to the experiment of refunds.

There is considerable theoretical literature on the effect of seed money, and most of them are broadly divided into two types. The first focuses on the effect of seed money to shift the minimal cost of project success. It considers that since the threshold costs for project success decreases due to seed money, the total contribution increases. We call this effect *the threshold shift effect* of seed money. This effect is only available to consider capital campaigns with fixed project costs. Studies based on a discrete public good provision model such as Andreoni (1998), Menezes *et al.* (2001), and Bac and Bag (2003) constitute this type. The second type of research focuses on the uncertainty of potential donors about the quality of charity. It explains that the existence of seed money credits the high quality of charity and increases the contribution from donors. We call this effect *the quality signal effect* of seed money. Vesterlund (2003) and Andreoni (2006a) are examples of such studies. The explanation by the quality signal effect has an advantage since it is available not only to capital campaigns but continuing campaigns without any fixed costs.[8]

Our model demonstrates that by an exclusive consideration of the threshold shift effect of seed money, the proportion of donors and the total amount of donation strictly and continuously increase to match the amount of seed money. This result is quite compatible with the field experimental study of List and Lucking-Reiley (2002), and supports the view that charitable giving actually involves a coordination game. Indeed, we can construct a model in which the threshold shift effect and the quality signal effect are complementary explanations for the effect of seed money. Our coordination game model and the results are meaningful even if we take the quality signal effect of seed money into account.

1.3 The Structure of the Paper

The structure of this paper is follows. In Section 2, we examine the basic model of charitable giving without seed money. In 2.1, we construct a model based on a simple global coordination game. In 2.2, we derive the equilibrium. In 2.3, we see the conditions for efficiency of agents and project success in this model. In Section 3, we consider the effect of seed money. In 3.1, we introduce seed money with the threshold shift effect, and see the impact on the behavior of potential donors. In 3.2, we calculate the minimal seed money for both project efficiency and success. In Section 4, we discuss the still remaining inconsistency with List and Lucking-Reiley (2002) and conceivable solutions for this inconsistency. Section 5 concludes. All proofs

[8]There are other types of explanations about the effect of seed money. Romano and Yildirim (2001) show that in a two stage game of a continuing public good provision, due to the warm-glow property itself, the announcement of the contributions in the first stage sometimes brings larger total contributions. Bag and Roy (2008) explain it by the uncertainty for other donors' valuations about the project.

of propositions and remarks are in the Appendix.

2 Basic Model and Results

In this section, we present a model of charitable giving that becomes a foundation for further comparative static analysis. The effect of seed money is not considered yet. We present the detailed game structure in 2.1, derive a unique equilibrium in 2.2, and provide remarks on the efficiency and project success in 2.3.

2.1 Preliminaries

There is a charity project and $n \in \mathbb{N}$ potential donors (agents). The project has a nonconvex technology. There exists a threshold level of its cost $C \in \mathbb{R}_{++}$, and if the total provision from agents exceeds the threshold C, then the quality of the project is significantly improved. Imagine, for example, a charitable campaign to promote child health care in a rural area of a developing country. If the charity collects sufficient donation to build a hospital, the quality of the project would rise notably. We call the project a *success* (in the ex-post sense) if the collected donation from agents is enough to cover C, and a *failure* (in the ex-post sense) if the total donation is less than C.

We consider cases wherein the number of agents is adequately large.[9] We adopt the assumption of a continuum of agents. To simplify the analysis, we normalize the length of the continuum of agents to 1. Let $c = \frac{C}{n}$. Under the normalized setting, c plays a role as the threshold cost instead of C. Thus we call c the threshold cost as well.

The charity announces the threshold cost c and solicits agents for donation. Through the announcement, each agent receives a signal of how worth the project is in both cases of project success and failure. Based on this signal, each agent makes a decision on whether to donate or not. To simplify the analysis, we assume that her decision is merely a choice from her action set $\{Donate, Not\ Donate\}$, and the amount of donation on the basis of her signal is given exogenously.[10]

Let $v_i \in \mathbb{R}$ be the signal of the project value for an agent i in the case of project success (ex-post sense). $v_i > 0$ means that i thinks the project value is positive and it is worth donating a positive amount of money if it

[9] In each field experiment of List and Lucking-Reiley (2002), the number of potential donors (people who took solicitation letters) was 500.

[10] It is widely adopted as a simplification technique to limit the elements of each agent's action set binary in literatures of global games. See, for example, Morris and Shin (2003). Moreover, even in the classical models of a discrete public good provision such as Palfrey and Rosenthal (1984) and Gradstein (1994), the elements of action set are limited binary to reduce complexities.

succeeds. $v_i \leq 0$ means that she thinks the project is not worthwhile and donating to it would be wasteful. In the case of $v_i \leq 0$, consider that the absolute value $|v_i|$ represents the degree of dislike for the project.

Assume that if $v_i > 0$, she donates $x(v_i)$ when she makes a decision *Donate*, where $x : \mathbb{R}_+ \to \mathbb{R}_+$ is the same function among all agents, and possesses the properties of twice differentiability, and for all $v > 0$, $0 < x(v) < v$, $x'(v) > 0$ and $x''(v) \leq 0$. (*i.e.*, $x(\cdot)$ is a weakly concave function.) Furthermore, assume that $\lim_{v \to 0} x(v) = 0$, $\lim_{v \to +\infty} x(v) = +\infty$ and there exists $\alpha \in (0, 1)$ such that $\lim_{v \to 0} x'(v) = \alpha$. Note that $v(\cdot)$ represents a wide class of functions including the constant function $\overline{x} : \mathbb{R}_+ \to \mathbb{R}_+$ such that for all $v > 0$, $\overline{x}(v) = \alpha v$.

Moreover, assume that if $v_i \leq 0$, she donates $\epsilon > 0$ if she decides to *Donate*. In the case of $v_i \leq 0$, it is natural to consider that in equilibria, she never decides to donate. The amount $\epsilon > 0$ is a technical assumption.

We assume throughout the paper that if the project is a failure, any agent i thinks that in the ex-post, the project is not attractive and the amount of donation with a minus sign itself is her payoff. Also assume that any agent has zero payoff if she does not donate, regardless of whether the project is a success or a failure.[11]

The payoff tables below summarises them.

If $v_i > 0$,

	Success	Failure
Donate	$v_i - x(v_i)$	$-x(v_i)$
Not Donate	0	0

If $v_i \leq 0$,

	Success	Failure
Donate	$v_i - \epsilon$	$-\epsilon$
Not Donate	0	0

Based on these ex-post payoff tables, each agent is assumed to calculate the von Neumann-Morgenstern expected interim payoff of her own, and decide whether she donates or not.

Let us introduce the signal distribution structure and information setting for agents. Assume that the signal for each agent v_i follows a normal distribution with mean θ and standard deviation σ independently and identically. Since the density function for a normal distribution approximates a wide class of symmetric density functions with a peak, this assumption is adequate.[12] Let $F(\cdot; \theta)$ denote the normal cumulative distribution function

[11] This assumption is based on the warm-glow property discussed in the Introduction.

[12] In the field experiments of List and Lucking-Reiley (2002), all potential donors met two criteria: (i) the household's annual income was above \$70,000, and (ii) the household was known to have previously given to a charity. In this case, the standard deviation σ is relatively small.

with mean θ and standard deviation σ, and $f(\cdot;\theta)$ denote its probability density function.

Following Morris and Shin (2003), we assume that it is common knowledge among agents that each signal v_i for an agent i is independently, identically, and normally distributed with standard deviation σ. However, each agent i has no information about the mean θ. She has to predict the mean θ on the basis of her realized signal v_i. We assume that her prior belief for θ is uniformly distributed over the real line $\mathbb{R}$. This prior belief is improper since the total probability mass is infinite, however, the posterior belief through Bayesian estimation is well defined.[13] She believes based on her signal v_i that θ follows the normal distribution with mean v_i and standard deviation σ. This is to say, from her point of view, $F(\cdot;v_i)$ is the cumulative distribution function of the actual mean θ, and $f(\cdot;v_i)$ is its probability density function. It is an interesting aspect of this improper uniform prior belief that the same function is available to represent both the distribution of real signals and the distribution of predictable mean based on belief of one agent.

In comparison with a standard Bayesian game wherein the signal distribution is common knowledge among agents, the assumption that the posterior belief of signal distribution for each agent is slightly different from those of others is quite natural. This is because, first, in a large economy where a continuum can approximate the total agents, if the distribution of signals is common knowledge, it almost represents a complete information game. It appears unrealistic. Second, especially when agents are relatively homogeneous (*i.e.*, σ is small) as in the field experiments of List and Lucking-Reiley (2002), it is reasonable that each agent predicts the signal distribution on the basis of her own signal.

A strategy for an agent i is a function mapping her receiving signal v_i to her action set $\{$*Donate*, *Not Donate*$\}$. If both actions occasion the equivalent expected payoff for i, we assume she selects *Not Donate*. This assumption is just to simplify the description, and the opposite assumption is of course available.

At this point, a Bayesian type of incomplete information game is defined. In the next subsection, we consider the equilibrium in this game.

2.2 The Equilibrium

We are interested in Bayesian Nash equilibria of the game defined in the previous subsection. If possible, a unique equilibrium is preferable especially for the purpose of comparative statics. The global game analysis makes it possible to provide a unique equilibrium with a few natural additional assumptions to the strategies of agents. Furthermore, it is the strategy profile surviving the iterated elimination of interim strictly dominated strategies.

[13]See Morris and Shin (2003) for a detailed explanation. They discuss this improper uniform prior belief with respect to the philosophy of Laplace.

Since this concept does not rely on the assumption that the strategies of other agents are common knowledge, it is a weaker condition than Bayesian Nash equilibrium.

In this subsection, we characterize a unique strategy profile by the iterated elimination of interim strictly dominated strategies as the equilibrium. The equilibrium strategy possesses two interesting properties: symmetry and threshold properties. A strategy is symmetric if all agents follow the same strategy. A strategy is a threshold strategy around some cutoff point $k \in \mathbb{R}$ if there exists a unique cutoff point k such that if her receiving signal v_i is above k, she donates and otherwise, she does not donate. A symmetric threshold strategy $s : \mathbb{R} \to \{\textit{Donate, Not Donate}\}$ is formally written as follows.

$$s(v_i) = \begin{cases} \textit{Donate} & \text{if } v_i > k \\ \textit{Not Donate} & \text{if } v_i \leq k \end{cases}$$

Note that symmetry and threshold properties of the equilibrium are not the assumptions but are obtained endogenously.

We introduce several functions that play important roles in the following investigation.

$$\int_k^{\infty} f(v;\theta)dv \tag{1}$$

$$\int_k^{\infty} x(v)f(v;\theta)dv \tag{2}$$

(1) represents the proportion of agents whose signal is larger than k when the mean of signals is θ. This function is useful no matter what strategies are adopted. (2) represents the total donation when the mean of signals is θ and any agent receiving a signal larger than the cutoff point k decides to donate. (2) is meaningful only when the threshold strategy is adopted by any agent.

At the beginning of the investigation, notice the remark below.

Remark 1. For any agent i with $v_i < 0$, *Not Donate* is the strictly dominant strategy.

This remark is straightforward from the payoff tables of the previous section, and has an important role in the global game analysis. We need a similar property that for an agent i with a large enough signal v_i, *Donate* is the strictly dominant strategy.

Remember that we assume the number of agents to be large and approximated by a continuum of agents with length 1. Therefore, an agent regards her contribution itself as zero even if she donates a positive amount of money. However, it is not natural that an agent with so large a signal that her own contribution covers the whole project cost C (*i.e.*, an agent i with signal $v_i > 0$ such that $x(v_i) > C$) would consider her contribution to be zero. Notice that it is not normalized cost c but the original C. Since her actual contribution covers the whole cost C, she has no reason to fear the project failure.

Assumption 1 below represents the above discussion. Let $V \in \mathbb{R}_{++}$ be such that $x(V) = C$.

Assumption 1. For any agent i with $v_i > V$, *Donate* is the strictly dominant strategy.

Note that the interim payoff of an agent i with a positive signal $v_i > 0$ is represented by

$$\Pr(\text{Success})(v_i - x(v_i)) + (1 - \Pr(\text{Success}))(-x(v_i)). \tag{3}$$

Thus, agent i decides to donate if

$$\begin{aligned} &(3) > 0 \\ \iff &\Pr(\text{Success}) > \frac{x(v_i)}{v_i}. \end{aligned}$$

Since $\lim_{v_i \to 0} x(v_i) = 0$ and $\lim_{v_i \to 0} x'(v_i) = \alpha$, we have

$$\lim_{v_i \to 0} \frac{x(v_i)}{v_i} = \alpha \qquad \text{(by l'Hospital's rule.)}$$

Let a function $p : \mathbb{R}_+ \to \mathbb{R}_{++}$ be such that

$$p(v_i) = \begin{cases} \frac{x(v_i)}{v_i} & \text{if } v_i > 0 \\ \alpha & \text{if } v_i = 0. \end{cases}$$

$p(v_i)$ represents the threshold probability such that i with $v_i > 0$ considers if the probability of project success is larger than $p(v_i)$, *Donate* leads to a higher expected payoff than *Not Donate*. On the other hand, if the probability of project success is less than $p(v_i)$, *Not Donate* guarantees a higher expected payoff than *Donate*. The reason that $p(0)$ is defined as the limit value at $v_i \to 0$ is just to simplify the following descriptions. Since $x(\cdot)$ is weakly concave and differentiable in $v_i > 0$, we have a remark below.

Remark 2. $p(\cdot)$ is weakly decreasing and continuous in $v_i \geq 0$.

[Figure 1 enters around here.]

Let $\hat{\theta} : \mathbb{R}_+ \to \mathbb{R}$ be a function such that for $v_i \geq 0$,

$$\int_{\hat{\theta}(v_i)}^{\infty} f(v; v_i) dv = p(v_i).[14]$$

[14] In the explicit form, $\hat{\theta}(v_i) = F^{-1}(1 - p(v_i); v_i)$, where $F^{-1}(\cdot; v_i)$ is the inverse function of $F(\cdot; v_i)$. This expression is rarely used in the following investigation.

See Figure 1 for the illustration. Remember that an agent i with a signal v_i believes the mean of normally distributed signals to be v_i itself. Thus the equation represents that i considers the probability of that the mean of signal distribution to be larger than $\hat{\theta}(v_i)$ is $p(v_i)$. (*i.e.*, i considers $\Pr(\theta \geq \theta(v_i)) = p(v_i)$.) Note that since $p(0) = \alpha \in (0, 1)$, $\hat{\theta}(0) \in \mathbb{R}$. Since $p(\cdot)$ is weakly decreasing in $v_i \geq 0$, we have the remark below.

Remark 3. (i) $\hat{\theta}(v_i)$ is strictly increasing and continuous in $v_i \geq 0$. (ii) For $v_i' > v_i \geq 0$, $\hat{\theta}(v_i') - \hat{\theta}(v_i) \geq v_i' - v_i$. (iii) $\lim_{v \to +\infty} \hat{\theta}(v) = +\infty$.

We introduce two more functions and discuss their properties.

$$\int_k^\infty x(v) f(v; \hat{\theta}(v_i)) dv \tag{4}$$

(4) is a special case of (2), and the mean of signals is $\hat{\theta}(v_i)$. Since $\hat{\theta}(v_i)$ is strictly increasing in $v_i \geq 0$, the following remark is straightforward.

Remark 4. (4) is strictly increasing and continuous in $v_i \geq 0$ and strictly decreasing and continuous in $k \geq 0$.

$$\int_{v_i}^\infty x(v) f(v; \hat{\theta}(v_i)) dv \tag{5}$$

(5) is a special case of (4). The mean of signals is $\hat{\theta}(v_i)$ and the cutoff point $k = v_i$. Remark 3 induces the next remark.

Remark 5. (5) is strictly increasing and continuous in $v_i \geq 0$.

Before characterizing the equilibrium, we add one more assumptions.

Assumption 2. $\int_V^\infty x(v) f(v; \hat{\theta}(V)) dv \geq c$

Assumption 2 states that if the mean of signals is $\hat{\theta}(V)$ and agents with signals more than V donate, the total donation from agents with signals more than V is larger than c and the project is a success. As already discussed when we introduced Assumption 1, the signal V is so large that $x(V) = C$. Since $\hat{\theta}(V)$ is also large in accordance with V, it is also a natural assumption.

Let G denote the Bayesian type of incomplete information game defined in Subsection 2.1 with Assumption 1 and 2. Now, we characterize a cutoff point and the equilibrium. Given a threshold cost c, let

$$k^* = \begin{cases} 0 & \text{if } \int_0^\infty x(v) f(v; \hat{\theta}(0)) dv \geq c \\ \kappa & \text{otherwise,} \end{cases}$$

where $\kappa \in \mathbb{R}_{++}$ satisfies

$$\int_\kappa^\infty x(v) f(v; \hat{\theta}(\kappa)) dv = c.$$

Notice that since (5) diverges to infinity when v_i goes to infinity (*i.e.*, (5) $\to +\infty$ ($v_i \to +\infty$)), κ is uniquely determined if $\int_0^\infty x(v) f(v; \hat{\theta}(0)) dv < c$.

Proposition 1. *Let a symmetric threshold strategy $s^*(\cdot)$ be such that*

$$s^*(v_i) = \begin{cases} \textit{Donate} & \textit{if } v_i > k^* \\ \textit{Not Donate} & \textit{if } v_i \leq k^*. \end{cases}$$

Then, in a game G, $s^*(\cdot)$ constitutes a unique strategy profile surviving the iterated elimination of interim strictly dominated strategies.

Note that from the definition of k^*, the actual mean of signal distribution θ has no effect in determining the equilibrium cutoff point k^*. In contrast, $\hat{\theta}(\cdot)$ is quite important in determining k^*. The example below illustrates the proposition.

Example 1. Let a threshold level of cost $c = 6$.[15] Let a standard deviation of signal distribution $\sigma = 10$. Let for all $v_i > 0$, $x(v_i) = \frac{1}{2}v_i$. Then, for all $v_i > 0$, $p(v_i) = \frac{1}{2}$ and $\hat{\theta}(v_i) = v_i$. We calculate $k^* = 4.02115$, and have Table 2.

[Table 2 enters around here.]

Note that the proportion of actual contributors is represented by

$$\int_{k^*}^{\infty} f(v;\theta)dv \tag{6}$$

and the total amount of donation is represented by

$$\int_{k^*}^{\infty} x(v)f(v;\theta)dv. \tag{7}$$

The mean amount of donation from actual contributors is (7)/(6), and the project is a success if (7) is larger than c.

As in Example 1, the equilibrium cutoff point k^* is often larger than 0. It is due to the particular assumption of global game that each agent possesses a different belief about the signal distribution. Under a standard Bayesian game model where the signal distribution and strategies of other agents constitute common knowledge, all agents with signals larger than 0 together decide whether to donate or not depending on the actual mean of signal θ in an equilibrium. Under the same setting as in Example 1, the standard Bayesian game analysis provides us that in an equilibrium, if $\theta > 4.02115$, agents with signals larger than 0 together decide to donate, and if $\theta \leq 4.02115$, any agent does not donate and the total amount of donation is equal to zero. It is a quite incompatible prediction with empirical research such as List and Lucking-Reiley (2002). It suggests the effectiveness of global game analysis in studies of charitable giving.

[15] In each field experiment of List and Lucking-Reiley (2002), the threshold level of project cost for success is \$3,000 and the number of potential donors is 500. (*i.e.*, $C = 3,000$ and $n = 500$.) In this case, $c = C/n = 6$.

2.3 Conditions for Efficiency and Success

In this subsection, we first consider the condition that achieves the ex-post Pareto efficiency of the donation for agents. Next, we consider the condition for project success.

We call the project is *efficient* if in the equilibrium, the action of agents constitutes the ex-post Pareto efficient. Formally, it is represented by that if $\int_0^\infty x(v)f(v;\theta)dv \geq c$, any agent i with signal $v_i > 0$ chooses *Donate* and any agent j with signal $v_j \leq 0$ chooses *Not Donate*, and otherwise, all agents select *Not Donate.* It is achievable in a equilibrium of the standard Bayesian Game discussed in the last paragraph of the previous subsection. However, under the more realistic global game setting, it is hardly obtained. In a game G, the following proposition holds.

Proposition 2. *A project is efficient if and only if* $k^* = 0$ *and* $\int_0^\infty x(v)f(v;\theta)dv \geq c$.

Actually, it is equivalent to the condition for project success when $k^* = 0$. The intuition of this proposition is quite simple. Even though the project is a failure, some agents with signals larger than k^* choose *Donate.* It implies that the project is not efficient. If the project is a success and $k^* > 0$, an agent i with $0 < v_i < k^*$ chooses *Not Donate.* It brings inefficiency. Therefore, the condition that occasions efficiency is only described in Proposition 2.

As already mentioned in the previous subsection, a project is a success if

$$\int_{k^*}^\infty x(v)f(v;\theta)dv \geq c. \tag{8}$$

It may be the most important matter for a charity fund-raiser. Note that if a project is efficient, then the project is a success. Besides, even in the case of $k^* > 0$, the project is a success as the proposition below states.

Proposition 3. *A project is a success if and only if it is efficient or* $\theta \geq \hat{\theta}(k^*)$.

If $k^* > 0$, $\int_{k^*}^\infty x(v)f(v;\hat{\theta}(k^*))dv = c$. Thus $\theta \geq \hat{\theta}(k^*)$ induces $\int_{k^*}^\infty x(v)f(v;\theta)dv \geq c$ and the project succeeds.

In the next section, we introduce seed money. The main purpose to introduce seed money is to obtain project success or efficiency. We show how to calculate the minimal amount of seed money for both project efficiency and success.

3 Seed Money

We introduce seed money exogenously. We see the threshold shift effect of the seed money to the donation behavior of agents in 3.1. We also see how to obtain the project success and efficiency by announcing seed money in 3.2.

3.1 The Threshold Shift Effect

Let $L \in [0, C)$ denote the amount of seed money. $L = 0$ represents the state wherein no seed money is granted. If seed money L is granted, the threshold level for project success shifts from C to $C - L$. Assume that notified information about the project never changes except the existence of seed money and its amount. We consider that all other structures of the game except the shift of threshold for the project success remain the same.

Let ℓ denote a normalized version of seed money, *i.e.*, $\ell = \frac{L}{n}$. Consequently, the normalized version of the shifted threshold is $c - \ell$. We also rewrite the additional assumptions in Subsection 2.2 as follows. Let $V(\ell) \in \mathbb{R}_{++}$ be such that $x(V(\ell)) = C - L$.

Assumption 1'. For any agent i with $v_i > V(\ell)$, *Donate* is the strictly dominant strategy.

Assumption 2'. $\int_{V(\ell)}^{\infty} x(v) f(v; \hat{\theta}(V(\ell)))dv \geq c - \ell$.

Let $G(\ell)$ denote a Bayesian game with the same structure as that in Subsection 2.1 with the shift of threshold level from c to $c - \ell$ and Assumptions 1' and 2'. Then we derive a unique equilibrium strategy $s^*(\cdot; \ell)$ in a game $G(\ell)$ as a corollary of Proposition 1.

Corollary 1. *In a game $G(\ell)$, a symmetric threshold strategy $s^*(\cdot; \ell)$ such that*

$$s^*(v_i; \ell) = \begin{cases} \textit{Donate} & \textit{if } v_i > k^*(\ell) \\ \textit{Not Donate} & \textit{if } v_i \leq k^*(\ell), \end{cases}$$

where

$$k^*(\ell) = \begin{cases} 0 & \textit{if } \int_0^{\infty} x(v) f(v; \hat{\theta}(0))dv \geq c - \ell \\ \kappa(\ell) & \textit{otherwise,} \end{cases}$$

and $\kappa(\ell) \in \mathbb{R}_{++}$ is such that

$$\int_{\kappa(\ell)}^{\infty} x(v) f(v; \hat{\theta}(\kappa(\ell)))dv = c - \ell$$

constitutes a unique strategy profile surviving the iterated elimination of interim strictly dominated strategies.

Hereafter, we consider the case of $\int_0^{\infty} x(v) f(v; \hat{\theta}(0))dv < c$. Otherwise, not only project success but efficiency is obtained without seed money, and the introduction of seed money is redundant. Let $\underline{\ell} = c - \int_0^{\infty} x(v) f(v; \hat{\theta}(0))dv$. According to Corollary 1, we have the remark below.

Remark 6. $k^*(\ell)$ is strictly decreasing and continuous in $\ell < \underline{\ell}$, and $k^*(\ell') = 0$ for $\ell' \geq \underline{\ell}$.

From Remark 6, we derive a proposition below which is quite compatible with the field experiment of List and Lucking-Reiley (2002).

Proposition 4. *If a granted seed money ℓ strictly increases, then both the proportion of agents who decide to donate and the total amount of donation strictly increase as long as $\ell < \underline{\ell}$.*

The example below illustrates this proposition.

Example 2. Similarly to Example 1, let a threshold level of cost $c = 6$, a standard deviation of signal distribution $\sigma = 10$, and for all $v_i > 0$, $x(v_i) = \frac{1}{2}v_i$. Consequently, we have the results in Table 3.

[Table 3 enters around here.]

The mechanism behind Proposition 4 is quite simple. It only depends on Remark 6. If $k^*(\ell)$ decreases, it is obvious that the number of agents with signal larger than the cutoff point $k^*(\ell)$ strictly increases, and then the proportion of agents who decide to donate and the total amount of donation also strictly increase. As discussed in the last part of Subsection 2.2, the emergence of the cutoff point $k^*(\ell)$ strictly larger than 0 is due to the global game assumption that the actual signal distribution is not common knowledge but noisily observed and guessed by each agent i on the basis of her signal v_i. This realistic setting of the global game is the key to the derivation of Proposition 4.

3.2 Minimal Seed Money for Efficiency and Success

The main purpose for introducing seed money is to obtain the project efficiency or project success when they are not achieved without seed money. Especially from the view point of charity fund-raiser, attaining the project success is crucial. Even though we do not explicitly consider the cost to introduce seed money, charity fund-raisers actually require a considerable effort to collect seed money, and are better off if the amount of seed money for the project efficiency or success is as small as possible. In this subsection, we derive a minimal amount of seed money to attain both project efficiency and success.

First, consider the minimal seed money for efficiency. Note that without seed money, the project is not efficient if $k^*(0) > 0$ or $\int_0^\infty x(v)f(v;\theta)dv < c$. In this case, we have the following proposition.

Proposition 5. *The minimal amount of seed money for project efficiency $\overline{\ell}$ is such that*

$$\overline{\ell} = \begin{cases} c - \int_0^\infty x(v)f(v;\hat{\theta}(0))dv & \text{if } \theta \geq \hat{\theta}(0) \\ c - \int_0^\infty x(v)f(v;\theta)dv & \text{otherwise.} \end{cases}$$

This proposition is straightforward from Proposition 2. If $\theta \geq \hat{\theta}(0)$, the amount of seed money to guarantee that agents with signals larger than 0 choose *Donate* is sufficient. If $\theta < \hat{\theta}(0)$, we need enough seed money ℓ so that the shifted threshold $c - \ell$ is weakly smaller than the possible total amount. (*i.e.*, $c - \ell \leq \int_0^\infty x(v)f(v;\theta)dv$.)

Next, we consider the minimal seed money for project success. The project is not a success without seed money if it is not efficient and $\theta < \hat{\theta}(k^*(0))$. In this case, in order to achieve the project success, we have the following proposition.

Proposition 6. *The minimal amount of seed money for the project success ℓ^* is such that if $\hat{\theta}(0) \leq \theta < \hat{\theta}(k^*(0))$, $\hat{\theta}(k^*(\ell^*)) = \theta$, and if $\theta < \hat{\theta}(0)$, $\ell^* = c - \int_0^\infty x(v)f(v;\theta)dv$.*

Note that from Remarks 3 and 6, in the case of $\hat{\theta}(0) \leq \theta < \hat{\theta}(k^*(0))$, $\hat{\theta}(k^*(\ell))$ is continuously and strictly decreasing in $\ell \in [0, \overline{\ell}]$. Thus ℓ^* is uniquely determined. In this case, the minimal seed money for project success is strictly smaller than that for efficiency, which suits our intuition. In Example 2, in the case of $\theta = 0$, both $\overline{\ell} = \ell^* = 2.01058$. However, in the case of $\theta = 2$, $\overline{\ell} = 2.01058$ and $\ell^* = 1.01058$.

To calculate the minimal seed money for project efficiency and success, a charity fund-raiser needs to know the actual mean of the distribution θ. Indeed, to collect an actual signal distribution is quite difficult for a charity fund-raiser. It involves a considerable cost for preliminary surveys on the potential donors to near the actual mean θ, and it is very beneficial.[16]

4 Discussion

In the previous section, we establish Proposition 4 that is quite compatible with the empirical result of List and Lucking-Reiley (2002). In this section, we discuss the still remaining inconsistency of our model with List and Lucking-Reiley (2002). In our model, the per-capita amount of donation

[16] If agents know the actual signal distribution, efficiency is obtained in an equilibrium. Thus the question arises "if charity fund-raiser announces the mean of distribution based on preliminary surveys, isn't efficiency obtained?" The answer may be "No" since agents suspect that strategic manipulation by charity fund-raiser with regard to the mean takes place. Certain studies on global game construct models with posterior belief of an agent on the basis of both her private and publicly announced signals. See, for example, Morris and Shin (2002, 2003) and Angeletos *et al.* (2006).

strictly decreases along with the increase of seed money, whereas the per-capita amount of donation strictly increases corresponding to the increase of seed money in List and Lucking-Reiley (2002). We discuss where this inconsistency come from and how to solve it.

In the previous section, Example 2 demonstrates that the per-capita amount of donation is strictly decreasing in regard to the amount of seed money. The mechanism is quite simple. We assume that the amount of donation $x(v_i)$ of agent i with signal v_i is exogenously given and never changed to the amount of seed money. From Remark 6, if the amount of seed money increases, the threshold signal $k^*(\ell)$ decreases. Note that an agent who changes her action from *Not Donate* to *Donate* due to the increase of seed money has a smaller signal and a smaller donation than agents who choose to *Donate* from the beginning. It induces the per-capita amount of donation to decrease.

To remove this inconsistency in the extended version of our model, we need to increase $x(v_i)$ according to the increase of seed money. Two approaches are considerable for this purpose.

One is to render $x(v_i)$ endogenously determined as an optimal value corresponding to the probability of success. This relies on the assumption that the effect of seed money is only the threshold shift effect. Since the probability of success depends on the amount of donation obtained from other agents, this optimization problem is highly complicated. At present, we are not sure whether this approach makes sense or not.

The other approach entails the introduction of the effect of seed money as quality signal. The increase of $x(v_i)$ corresponding to the increase of seed money is considered to be the consequence of the quality signal effect of seed money. For example, consider the simple introduction of the quality signal effect as follows. Let θ be the mean of each agent's signal, similarly to Section 2. Let $\theta : [0, c) \to \mathbb{R}$ be a strictly increasing and continuous function such that $\theta(0) = \theta$. Assume that when seed money ℓ is granted, the mean of signal θ is shifted to $\theta(\ell)$. Moreover, assume that the standard deviation σ remains the same. We interpret this shift of θ to $\theta(\ell)$ as that an agent i receiving a signal v_i when no seed money is granted receives a signal $v_i(\ell) = v_i + (\theta(\ell) - \theta)$ when seed money ℓ is granted. It is obvious that in this model, the inconsistency about the per-capita amount of donation is resolved.[17]

The second explanation also suggests that we can construct a model in which the threshold shift effect and the quality signal effect of seed money are complementary. Which effect actually works is still an interesting question in both theoretical and empirical studies.

[17] An interesting aspect of the previous literature that considers the effect of seed money as quality signal such as Vesterlund (2003) and Andreoni (2006a) is that the contributors for seed money and the amount of seed money are determined endogenously.

5 Concluding Remark

In this paper, we construct a global coordination game model of charitable giving, and show that merely by considering the threshold effect of seed money, the proportion of agents that decides to donate and the total amount of donations strictly and continuously increase according to the increase of seed money.

We construct a simultaneous model to be compared with the empirical study of List and Lucking-Reiley (2002). However, the actual capital campaigns of charities usually allow agents to donate in a certain period, and may be represented by dynamic games. Even in the dynamic models, our view that the charitable donation is indeed a coordination game if potential donors possess the warm-glow properties in their preferences plays an important role.[18] We hope that this paper encourages the further studies of donation behavior and helps future charity projects.

Appendix

In the Appendix, we provide proofs of Propositions and Remarks. We omit proofs of Remarks 1 and 4 since they are straightforward.

Proof of Remark 2. The continuity and differentiability of $p(\cdot)$ are obvious. Note that for $v_i > 0$,

$$p'(v_i) = \frac{-x(v_i) + v_i x'(v_i)}{v_i^2}.$$

Since $x(\cdot)$ is weakly concave in $v_i > 0$, $-x(v_i) + v_i x'(v_i) \leq 0$. It implies that for $v_i > 0$, $p'(v_i) \leq 0$, and $p(v_i)$ is weakly decreasing in $v_i \geq 0$. $\square$

Proof of Remark 3. Remember that $\int_{\hat{\theta}(v_i)}^{\infty} f(v; v_i)dv = p(v_i)$ for $v_i > 0$. Since $p(\cdot)$ is weakly decreasing in $v_i \geq 0$ by Remark 2, for $v_i' > v_i > 0$,

$$\int_{\hat{\theta}(v_i)}^{\infty} f(v; v_i)dv = p(v_i) \geq p(v_i') = \int_{\hat{\theta}(v_i')}^{\infty} f(v; v_i')dv.$$

Thus the continuity of $p(\cdot)$ implies (i) $\hat{\theta}(v_i)$ is strictly increasing and continuous in $v_i \geq 0$.

Note that if $p(v_i) = p(v_i')$, $\hat{\theta}(v_i') - \hat{\theta}(v_i) = v_i' - v_i$ and if $p(v_i) > p(v_i')$, $\hat{\theta}(v_i') - \hat{\theta}(v_i) \geq v_i' - v_i$. (See Figure 2 for the illustration.) They imply (ii) for $v_i' > v_i \geq 0$, $\hat{\theta}(v_i') - \hat{\theta}(v_i) \geq v_i' - v_i$.

Since $\hat{\theta}(0) \in \mathbb{R}$, the construction of $\hat{\theta}(v_i)$ as $\int_{\hat{\theta}(v_i)}^{\infty} f(v; v_i)dv = p(v_i)$ implies (iii) $\lim_{v \to +\infty} \hat{\theta}(v) = +\infty$. $\square$

[18] Famous dynamic models of investments in projects include Admiti and Perry (1991) and Marx and Matthews (2000). Dynamic models of investments in projects in the global game framework are, for example, Heidhues and Melissas (2006) and Dasgupta (2007).

[Figure 2 enters around here.]

Proof of Remark 5. Let $v' > v > 0$. Since $\hat{\theta}(v_i') - \hat{\theta}(v_i) \geq v_i' - v_i$ by Remark 3, $\int_{v_i}^{\infty} f(v; \hat{\theta}(v_i))dv \leq \int_{v_i'}^{\infty} f(v; \hat{\theta}(v_i'))dv$. Since $x(\cdot)$ is a strictly increasing function, it implies that

$$\int_{v_i}^{\infty} x(v) f(v; \hat{\theta}(v_i))dv < \int_{v_i'}^{\infty} x(v) f(v; \hat{\theta}(v_i'))dv.$$

We have the statement of the remark. □

Proof of Proposition 1. Proposition 2.1 of Morris and Shin (2003) provides the equilibrium of a general global game where the (ex-post) payoff of an agent depends on her action, the proportion of other agents' strategies, and her own signal. Even though in our game G, the payoff of an agent depends on her action, the total amount of donation, and her own signal, our Proposition 1 is essentially the same as their Proposition 2.1. We owe Morris and Shin (2003) and originally Carlsson and van Damme (1993) their proof techniques.

Case 1: $\int_0^{\infty} x(v) f(v; \hat{\theta}(0))dv < c$.

(A) First, we construct a sequence of recursive values $\overline{v}^0, \overline{v}^1, \overline{v}^2, \cdots$ such that for agents with signals larger than each value, *Not Donate* is the interim strictly dominated strategy.

By Assumption 1, for any agent i with $v_i > V$, *Donate* is the strictly dominant strategy, and *Not Donate* is strictly dominated. Let $\overline{v}^0 = V$.

Let $t \in \mathbb{N} \cup \{0\}$. Assume as an induction hypothesis that for any agent i with $v_i > \overline{v}^t$, *Not Donate* is the interim strictly dominated strategy, and they choose to *Donate*. Let $\overline{v}^{t+1}$ be such that

$$\int_{\overline{v}^t}^{\infty} x(v) f(v; \hat{\theta}(\overline{v}^{t+1}))dv = c. \tag{9}$$

Suppose, at first, that all the other agents with signals smaller than $\overline{v}^t$ choose *Not Donate*. Then, (9) represents that if the mean of signals is $\hat{\theta}(\overline{v}^{t+1})$, the total amount of donation equals c. Thus if the mean of signals is larger than $\hat{\theta}(\overline{v}^{t+1})$, the project is a success.

Note that for an agent i with $v_i > \overline{v}^{t+1}$, $\Pr(\theta \geq \hat{\theta}(\overline{v}^{t+1})) > \Pr(\theta \geq \hat{\theta}(v_i)) = p(v_i)$ since $\hat{\theta}(v_i) > \hat{\theta}(\overline{v}^{t+1})$. It means that for i's point of view, the probability of success is larger than $p(v_i)$, and *Donate* is more profitable than *Not Donate*.

Notice that if we drop the supposition that all other agents with signals smaller than $\overline{v}^t$ choose *Not Donate*, the probability of success for i with $v_i > \overline{v}^{t+1}$ is higher than that with the supposition. Thus, for i with $v_i > \underline{v}^{t+1}$, *Not Donate* is the interim strictly dominated strategy.

Next, we show that $\kappa \leq \overline{v}^{t+1} \leq \underline{v}^t \leq V$.

First, notice $\kappa \leq V = \overline{v}^0$ by Assumption 2 and Remark 5.

Assume as induction hypothesis that $\kappa \leq \overline{v}^t$. Then $\int_\kappa^\infty x(v)f(v;\hat{\theta}(\kappa))dv = c$ by the definition of κ and Remark 5 imply that

$$\int_{\overline{v}^t}^\infty x(v)f(v;\hat{\theta}(\overline{v}^t))dv \geq c. \tag{10}$$

(9), (10) and Remark 4 imply that $\overline{v}^t \geq \overline{v}^{t+1}$. Similarly, since $\kappa \leq \overline{v}^t$, $\int_\kappa^\infty x(v)f(v;\hat{\theta}(\kappa))dv = c$, (9) and Remark 4 imply $\overline{v}^{t+1} \geq \kappa$. Hence $\kappa \leq \overline{v}^{t+1} \leq \underline{v}^t \leq V$.

By the recursive way, we have that the sequence $\overline{v}^0, \overline{v}^1, \overline{v}^2, \cdots$ is weakly decreasing and has an lower bound κ. Thus it has the limit value. Let $\overline{v}$ be the limit value. (*i.e.*, $\lim_{t\to+\infty} \overline{v}^t = \overline{v}$.) Since (4) is continuous in both k and v_i by Remark 4, we have that $\int_{\overline{v}}^\infty x(v)f(v;\hat{\theta}(\overline{v}))dv = c$. It implies $\overline{v} = \kappa$.

(B) Similarly, we can construct a sequence of recursive values $\underline{v}^0, \underline{v}^1, \underline{v}^2, \cdots$ such that for agents with signals smaller than each value, *Donate* is the interim strictly dominated strategy.

Let $\underline{v}^0 = 0$. Given $t \in \mathbb{N} \cup \{0\}$, let $\underline{v}^{t+1}$ be such that

$$\int_{\underline{v}^t}^\infty x(v)f(v;\hat{\theta}(\underline{v}^{t+1}))dv = c.$$

Then, Remark 1 and the symmetric reasoning of (A) guarantee that for agents with signals smaller than each value, *Donate* is the interim strictly dominated strategy.

Similarly to (A), we have that $0 \leq \underline{v}^t < \underline{v}^{t+1} < \kappa$ for any $t \in \mathbb{N} \cup \{0\}$, and $\lim_{t\to+\infty} \underline{v}^t = \kappa$.

By the discussions (A) and (B), we have the statement of Case 1.

Case 2: $\int_0^\infty x(v)f(v;\hat{\theta}(0))dv \geq c$.

(A) We can use the same reasoning of (A) in Case 1 with only one modification. Instead of (9), letting

$$\overline{v}^{t+1} = \begin{cases} 0 & \text{if } \int_{\overline{v}^t}^\infty x(v)f(v;\hat{\theta}(0))dv \geq c \\ w^{t+1} & \text{otherwise,} \end{cases}$$

where $w^{t+1} \in \mathbb{R}_+$ is such that $\int_{\overline{v}^t}^\infty x(v)f(v;\hat{\theta}(w^{t+1}))dv = c$.

Then we have that $0 \leq \overline{v}^{t+1} \leq \underline{v}^t \leq V$ for any $t \in \mathbb{N} \cup \{0\}$, and $\lim_{t\to+\infty} \overline{v}^t = 0$.

(B) Since Remark 1 says that for any agent i with $v_i < 0$, *Not Donate* is strictly dominant.

By the discussions (A) and (B), we have the statement for Case 2. $\square$

Proof of Proposition 2. The *if* part is obvious. The *only if* part is explained in the paragraph after this proposition. □

Proof of Proposition 3. *if* part : It is obvious that efficiency implies project success. The explanation for the case of $\theta \geq \hat{\theta}(k^*)$ is given in the paragraph after this proposition. *only if* part : We show the contraposition. Suppose $\theta < \hat{\theta}(k^*)$ and ($k^* > 0$ or $\int_0^\infty x(v)f(v;\theta)dv < c$). If $\int_0^\infty x(v)f(v;\theta)dv < c$, the project is never a success. If $\theta < \hat{\theta}(k^*)$ and $k^* > 0$, then $\int_{k^*}^\infty x(v)f(v;\hat{\theta}(k^*))dv = c > \int_{k^*}^\infty x(v)f(v;\theta)dv$. It induces the project failure. □

Proof of Remark 6. In the case of $\ell \geq \underline{\ell}$, $k^*(\ell) = 0$ is directly induced from Corollary 1. We consider the case of $\ell < \underline{\ell}$. Let a small $\epsilon > 0$. Then, from Corollary 1, we have

$$\int_{k^*(\ell)}^\infty x(v)f(v;\hat{\theta}(k^*(\ell)))dv = c - \ell \tag{11}$$

and

$$\int_{k^*(\ell+\epsilon)}^\infty x(v)f(v;\hat{\theta}(k^*(\ell+\epsilon)))dv = c - (\ell+\epsilon). \tag{12}$$

By subtracting both sides of (12) from those of (11), we have

$$\int_{k^*(\ell)}^\infty x(v)f(v;\hat{\theta}(k^*(\ell)))dv - \int_{k^*(\ell+\epsilon)}^\infty x(v)f(v;\hat{\theta}(k^*(\ell+\epsilon)))dv = \epsilon > 0. \tag{13}$$

Remark 5 and (13) imply $k^*(\ell) > k^*(\ell+\epsilon)$. Hence, $k^*(\ell)$ is strictly decreasing and continuous in $\ell < \underline{\ell}$. □

Proof of Proposition 4. Let $\ell < \underline{\ell}$. Note that, under this condition, the proportion of agents who decide to donate is represented by $\int_{k^*(\ell)}^\infty f(v;\theta)dv$ and the total amount of donation is given by $\int_{k^*(\ell)}^\infty x(v)f(v;\theta)dv$. Since both are strictly decreasing in $k^*(\ell)$, Remark 6 induces the statement. □

Proof of Proposition 5. First consider the case of $\theta \geq \hat{\theta}(0)$. Let $\ell \geq \overline{\ell}$. Then $\int_0^\infty x(v)f(v;\theta)dv \geq \int_0^\infty x(v)f(v;\hat{\theta}(0))dv \geq c - \ell$. It occasions the project success and $k^*(\ell) = 0$. Thus it is efficient. On the other hand, let $\ell' < \overline{\ell}$. Then $\int_0^\infty x(v)f(v;\hat{\theta}(0))dv = c - \overline{\ell} < c - \ell'$. Then $k^*(\ell') > 0$ and it brings inefficiency.

Next, we consider the other case, in which $\theta < \hat{\theta}(0)$. Let $\ell \geq \overline{\ell}$. Then $\int_0^\infty x(v)f(v;\hat{\theta}(0))dv \geq \int_0^\infty x(v)f(v;\theta)dv \geq c - \ell$. It occasions the project success and $k^*(\ell) = 0$. Thus it is efficient. Let $\ell' < \overline{\ell}$. Then $\int_0^\infty x(v)f(v;\theta)dv = c - \overline{\ell} < c - \ell'$. It occasions the project failure and inefficiency. □

Proof of Proposition 6. First consider the case of $\hat{\theta}(0) \leq \theta < \hat{\theta}(k^*(\ell^*))$. Let $\ell \geq \ell^*$. Remark 6 implies that $k^*(\ell) \leq k^*(\ell^*)$. Thus we have $\hat{\theta}(k^*(\ell)) \leq \hat{\theta}(k^*(\ell^*)) = \theta$. Since Corollary 1 states that

$$\int_{k^*(\ell)}^{\infty} x(v) f(v; \hat{\theta}(k^*(\ell))) dv = c - \ell,$$

$\hat{\theta}(k^*(\ell)) \leq \theta$ implies that

$$\int_{k^*(\ell)}^{\infty} x(v) f(v; \theta) dv \geq c - \ell.$$

It occasions the project success. Let $\ell' < \ell^*$. Then the symmetric reasoning brings us $\int_{k^*(\ell')}^{\infty} x(v) f(v; \theta) dv < c - \ell'$. It induces the project failure.

For the case of $\theta < \hat{\theta}(0)$, the proof is exactly the same as that for the case of $\theta < \hat{\theta}(0)$ in Proposition 5. □

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Table1: Results of the Field Experiment by List and Lucking-Reiley (2002)

A. Experimental Design			
Number of solicitations mailed	500	500	500
Seed money (%)	10%	33%	67%
Seed money ($)	$300	$1,000	$2,000
B. Result			
Number of contributions	17	33	42
Participation rate	3.40%	6.60%	8.40%
Total contributions	$202	$805	$1,485
Mean amount given	$11.88	$24.39	$35.36
Standard error of mean amount	$2.27	$2.50	$2.26

Source: Table 1 of List and Lucking-Reiley (2002)

Table 2: the Results of Example 1

θ	proportion of donors	total donation	mean amount of donation	success/failure
5	0.538988	6.66530	12.3663	s
0	0.343800	3.67958	10.7027	f
-5	0.183498	1.73830	9.47316	f
-10	0.080441	0.68844	8.55843	f

Table 3: the Results of Example 2

θ	ℓ	k^*	proportion of donors	total donation	mean amount of donation	success /failure
2	0	4.02115	0.419913	4.74859	11.30851	f
	1	2.02115	0.499156	4.98773	9.99233	f
	2	0.02115	0.578432	5.06894	8.76324	s
	3	0	0.57926	5.06895	8.75073	s
0	0	4.02115	0.343800	3.67958	10.7027	f
	1	2.02115	0.419913	3.90876	9.30850	f
	2	0.02115	0.499156	3.98941	7.99231	f
	3	0	0.500000	3.98942	7.97884	s

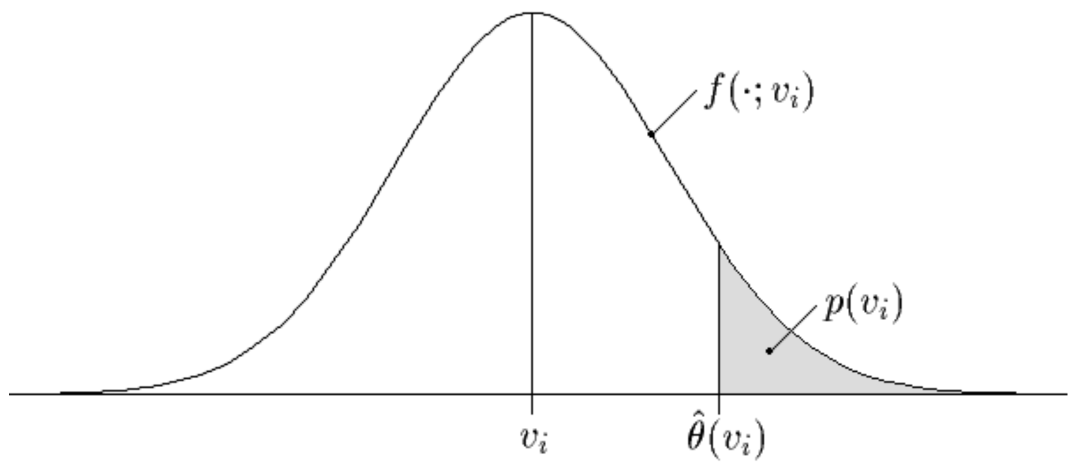

Figure 1: The relationship between v_i, $p(v_i)$, and $\hat{\theta}(v_i)$

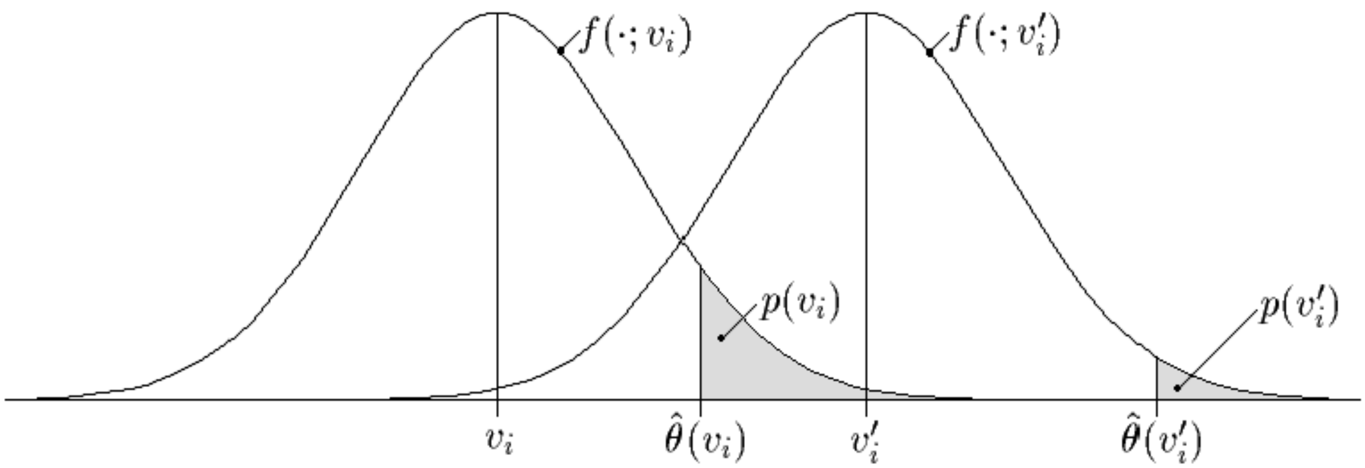

Figure 2: Illustration for the proof of Remark 3